Oblique Alfvén and Magnetosonic Solitons and Nonlinear Structures in Magnetized Plasma

N. Ya. Kotsarenko, Yu. G. Rapoport, A. A. Shvidkij*
and Yu. V. Khotyaintsev**
Astronomy and Space Physics Department,
Kyiv Shevchenko University, Glushkov pr. 6,
Kyiv 22, 252022 UKRAINE
*Now at University of Toledo, Toledo, USA
**Now at Swedish Institute of Space Physics, S-75591 Uppsala
rapoport@astrophys.ups.kiev.ua

June 2, 1998

Abstract

The problem of nonlinear waves and structures in magnetized plasma was investigated. Integrals of automodel equations with arbitrary amplitudes were obtained. Two possible types of solitons, compressive and rarefactive, and three types of structures are obtained for cold plasma approximation. A comparison of the exact solution of nonlinear problem and small amplitude approximation solution was performed. It is shown that for intermediate values of angles between wave vector and magnetic field the range of validity for small amplitude solution could be wider then it used to be excepted.

1 Introduction

First works devoted to the nonlinear theory of MHD waves in plasma appeared in sixties. For example, one-dimensional plasma motions perpendicular to magnetic field [1] and waves parallel to the magnetic field [2, 3, 4] were studied. Shock magnetosonic waves (MsW) and solitons were studied later in connection with an investigation of nonlinear phenomena near the border of a solar wind and the Earth's magnetosphere [5, 6], as a mechanism of the solar corona heating [7] and a possible mechanism of heating of the plasma in thermonuclear devices [8, 9]. The papers [1, 2, 3, 4] were devoted to the formed waves of large amplitude. Later on, and weak dispersion MsW and Alfvén waves of small amplitude [10, 11] became the main object of the theoretical investigation. Two-dimensional small amplitude MsW, described by Kadomtsev-Petviashvili equation [11] were also studied [12] with regard to the influence of particle collisions [11] and Landau damping [13]. It was shown that for the quasi parallel propagation MsW are governed by the modified Shrödinger equation with solutions in specific cases in the form of solitons of KdV and mKdV type [14]. A great number of theoretical and experimental studies was carried out to describe and observe various structures in plasma. For example, obliquely propagating ion-acoustic waves in magnetized plasma consisting of warm adiabatic ions and two types of Maxwellian electrons were studied in [15]. KdV equation was obtained, and the conical wave solution was discussed in this work. An upper-hybrid soliton of nonlinear Schrödinger type, when the pulse speed exceeds the magnetosonic speed, was obtained in [16]. The exact analytical solution of the nonlinear MHD equations for perturbations traveling across the external magnetic field was presented in [17].

Starting with Coleman [18], numerous observations of ultra low frequency (ULF) MHD fluctuations of the large amplitude in the space plasma were reported [19, 20]. Large amplitude disturbances of the magnetic field near the bow-shock due to the MsW were observed by the "Prognoz-10" satellite [21]. Alfvén waves in the Solar wind and comet plasma were discussed in [22].

In this work primary consideration is the exact solutions for the one-dimensional solitons and structures, however, we should mention the latest results concerning the multi-dimensional waves and structures. The results for two- and three- dimensional waves in magnetized plasma were presented in papers [23, 24, 25]. In general case, the dynamics of dispersive Alfvén waves of the finite amplitude is described by the system of coupled nonlinear equations [24, 25] with different types of nonlinearities including a vector nonlinearity. These three-dimensional equations could be used to study the modulation and filamentation instabilities as well as the envelope solitons. This model describes new plasma structures in comparison with the one-dimensional case, for example, vortex chains, solitary dipolar and monopolar vortex structures [26]. Such structures in cold plasma could help to explain the forms of active auroral arcs [28]. The development of this theory for the case of warm plasma was presented in [24]. Two-dimensional equation of the Kadomtsev-Petviashvili type for Alfvén and magnetosonic waves propagating at a small angle to the Earth's magnetic field was applied in [26] to explain the observations of the stable Alfvén-like solitons by Aureol 3 satellite. It was also suggested in [26] to apply this theory to interpret the chevron structures of the "pearls" that are occasionally observed in the Earth's magnetosphere.

The objectives of this work are:

1. To obtain the complete description of the one-dimensional ULF waves in the cold magnetized plasma;

2. To compare the results obtained by the Reductive Perturbation Method (RPM) and the results obtained from the exact solution;

3. To derive simple analytical description of the ULF waves in the cold plasma, suitable for the qualitative analysis.

2 Formulation of the problem

Let us consider cold collisionless non-relativistic plasma with one sort of ions. Such plasma is governed by the equations of two-fluid magnetohydrodynamics

$$\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \vec{\nabla})\vec{v}_e = -\frac{e}{m}\left(\vec{E} + \frac{\vec{v}_e}{c} \times \vec{H}\right) \quad (1)$$

$$\frac{\partial n_e}{\partial t} + div(n_e \vec{v}_e) = 0 \quad (2)$$

$$\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \vec{\nabla})\vec{v}_i = \frac{e}{M}\left(\vec{E} + \frac{\vec{v}_i}{c} \times \vec{H}\right) \quad (3)$$

$$\frac{\partial n_i}{\partial t} + div(n_i \vec{v}_i) = 0 \quad (4)$$

and by Maxwell equations

$$rot\vec{H} = -\frac{4\pi e}{c}(n_e \vec{v}_e - n_i \vec{v}_i) \quad (5)$$

$$rot\vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \quad (6)$$

$$div\vec{H} = 0 \quad (7)$$

Here m, $\vec{v}_e$, n_e and M, $\vec{v}_i$, n_i are masses, velocities and densities of electrons and ions respectively. The displacement current is neglected i n equation (5) what is valid for the considered low frequency.

Let us consider that the wave propagates along the z axis and the external magnetic field lies in xz plane at the angle θ to the z axis.

$$\vec{H} = \{H_0 \sin\theta, 0, H_0 \cos\theta\}$$

By introduction of automodel variable

$$\xi = z - v_0 t,$$

where v_0 is the wave velocity, the initial system of equations (1-7) is reduced to the system of ordinary differential equations. Full magnetic field is presented in the form

$$\vec{H} = \vec{H}_0 + \vec{h}(\xi).$$

It is possible to obtain the motion integral for the reduced system of automodel equations. From the equations (2) and (4) it follows that

$$n_{e,i} = \frac{n_0}{1 - \frac{v_{ze,i}}{v_0}}$$

Taking into account z component of eq. (5), one can obtain

$$v_{ez} = v_{iz} = v_{||}(\xi) \tag{8}$$

$$n_e = n_i = \frac{n_0}{1 - \frac{v_{||}}{v_0}} \tag{9}$$

while x and y component of eq. (5) give

$$\frac{dh_y}{d\xi} = \frac{4\pi e}{c} n(v_{ex} - v_{ix}), \frac{dh_x}{d\xi} = -\frac{4\pi e}{c} n(v_{ey} - v_{iy}) \tag{10}$$

From the eq. (6) we obtain

$$E_x = \frac{v_0}{c} h_y, E_y = -\frac{v_0}{c} h_x \tag{11}$$

z components of the motion equations (1) and (3) are

$$m(v_{||} - v_0)\frac{dv_{||}}{d\xi} = -e\left(E_z + \frac{v_{ex}h_y}{c} - \frac{v_{ey}}{c} H_0 sin\theta - \frac{v_{ey}h_x}{c}\right)$$

and

$$M(v_{||} - v_0)\frac{dv_{||}}{d\xi} = e\left(E_z + \frac{v_{ix}h_y}{c} - \frac{v_{iy}}{c} H_0 sin\theta - \frac{v_{iy}h_x}{c}\right).$$

Adding these two equations and taking into account eq. (10) we obtain:

$$v_{||} = \frac{1}{8\pi n_0 (M+m) v_0}(h_x^2 + h_y^2 + 2H_0 sin\theta h_x) \tag{12}$$

x and y components of the motion equation for the electrons (1) are:

$$(v_0 - v_{||})\frac{dv_{ex}}{d\xi} - \frac{e}{mc}(v_0 - v_{||})h_y - v_{ey}\omega_{He} cos\theta = 0$$

and

$$(v_0 - v_{||})\frac{dv_{ey}}{d\xi} + \frac{e}{mc}(v_0 - v_{||})h_y - v_{||}\omega_{He} sin\theta + v_{ex}\omega_{He} cos\theta = 0.$$

Assuming waves scale $L \gg \frac{v_0}{\omega_{He}}$, it is possible to neglect $\frac{d}{d\xi}$ (this is valid if the angle to the magnetic field is not very close to $\frac{\pi}{2}$). Then we have

$$v_{ey} = v_{||} tg\theta - (v_0 - v_{||})\frac{h_x}{H_0 cos\theta}, v_{ex} = -(v_0 - v_{||})\frac{h_y}{H_0 cos\theta}. \tag{13}$$

Using x and y components of the ions motion equation and eq. (10) and (13) we finally obtain the following system of equations:

$$\left(1 - \frac{v_{||}}{v_0}\right) cos\theta \frac{d\tilde{h}_x}{dx} = \left(1 - \frac{v_{||}}{v_0} - \beta cos^2\theta\right) \tilde{h}_y \tag{14}$$

$$\left(1 - \frac{v_{||}}{v_0}\right) cos\theta \frac{d\tilde{h}_y}{dx} = -\left(1 - \frac{v_{||}}{v_0} - \beta cos^2\theta\right) \tilde{h}_x + \frac{v_{||}}{v_0} sin\theta \tag{15}$$

$$\frac{v_{||}}{v_0} = \beta\frac{\tilde{h}_x^2 + \tilde{h}_y^2}{2} + \beta sin\theta \tilde{h}_x \tag{16}$$

where $\tilde{h}_{x,y} = \frac{h_{x,y}}{H_0}$, $x = \frac{\omega_{Hi} v_0}{C_A^2}\xi$, $C_A = \frac{H_0}{\sqrt{4\pi n_0(M+m)}}$ is Alfvén velocity, $\beta = \frac{C_A^2}{v_0^2}$. It is useful to introduce $u = 1 - \beta cos^2\theta$ and $\varphi = \frac{v_{||}}{v_0}$. Division of eq. (14) by eq. (15) yields the integral

$$-\frac{1}{2}(u - \varphi)^2 = \beta sin\theta u \tilde{h}_x + C \tag{17}$$

where C is the integrating constant. Integral (17) can also be used in the form with magnetic field components presented in their direct form. After introduction $\alpha = 2(cos^2\theta - \frac{1}{\beta})$, we obtain:

$$(\tilde{h}_x^2 + \tilde{h}_y^2 + 2\tilde{h}_x sin\theta + \alpha)^2 = 4\alpha\tilde{h}_x sin\theta + C \tag{18}$$

3 The types of waves and the regions of their existence

The general view of the phase plane ($\tilde{h}_y$ versus $\tilde{h}_x$) obtained using integral (17) is presented in Fig.1a. Different curves correspond to different values of parameter $i = c - \alpha^2$ respectively to fig. 1b. As seen from (18), for solitary wave, while $|\xi| \to \infty$, $\tilde{h}_x, \tilde{h}_y \to 0$, integrating constant $c = \alpha^2$ and $i = 0$. Soliton solutions represent two separatrixes on the phase plane. These separatrixes separate regions of different periodical solutions: the region outside outer separatrix corresponds to type (a) structures, the outer separatrix - compressive soliton, the region between separatrixes - type (b) structures, inner separatrix - rarefactive soliton, region inside inner separatrix - type (c) structures. Solitary solutions are discussed in 3.1,3.2, periodical solutions are discussed in 3.3.

3.1 Solitary waves

From the condition while $|\xi| \to \infty$, $\tilde{h}_x \to 0$ we can find the constant C in the integral (10) $c = -\frac{u^2}{2}$. Substituting C into eq. (17) we obtain

$$(u - \varphi)^2 = u^2 - 2\beta sin\theta u\tilde{h}_x \tag{19}$$

or in direct form

$$\tilde{h}_y = \pm\sqrt{\frac{2u}{\beta} - \tilde{h}_x - 2\beta sin\theta\tilde{h}_x \pm \frac{2}{\beta}\sqrt{u^2 - 2\beta sin\theta u\tilde{h}_x}} \tag{20}$$

Eq. (19) enables us to get to the following conclusions:

The asymptotic at $|\xi| \to \infty$ is governed by equation (19) with the sign "-" under the root and $\tilde{h}_y \to 0$, while $\tilde{h}_x \to 0$ only in this case. In the case of the sign "+" $\tilde{h}_y \to \pm\sqrt{\frac{u}{\beta}}$. Considering asymptotic $\tilde{h}_x, \tilde{h}_y \to 0$, the condition of existence of solitary waves can be found:

$$1 < \beta < \frac{1}{cos^2\theta} \tag{21}$$

or

$$C_A cos\theta < v_0 < C_A \tag{22}$$

As could be seen from fig. 1a, two solitary waves exist. The first one corresponds to the internal separatrix, the second corresponds to the outer separatrix. To integrate system (14-16), $\tilde{h}_{x,y}$ has to be expressed in terms of φ:

$$\tilde{h}_x = -\frac{(\varphi - 2u)}{2u\beta sin\theta},$$

$$\tilde{h}_x = \frac{\varphi}{2u\beta sin\theta}\sqrt{2u\beta sin^2\theta - (\varphi - 2u)^2}.$$

Then using eq. (14-16) we can obtain the equation for the function φ:

$$2cos\theta\frac{d\varphi}{dx} = \frac{\varphi}{1-\varphi}\sqrt{4u\beta sin^2\theta - (\varphi - 2u)^2} \tag{23}$$

As could be seen from eq.(23), the condition of positiveness of the discriminant is $-a \leq \varphi - 2u \leq a$, where $a = 2sin\theta\sqrt{u\beta}$. By this means the amplitude φ is confined in definite limits: $\varphi_{min} \leq \varphi \leq \varphi_{max}$, where $\varphi_{min} = (a - 2u) < 0$ and $\varphi_{max} = a + 2u$. It follows from the relation $n = \frac{n_0}{(1-\varphi)}$ that the condition $0 < \varphi \leq \varphi_{max}$ corresponds to a compressive wave, and the condition $\varphi_{min} \leq \varphi < 0$ corresponds to a rarefactive wave. As a result of eq.(23) integrating, we finally obtain for the compressive and rarefactive waves respectively:

$$\varphi = \frac{a-2u}{\frac{2a}{a+2u}ch^2\left(\sqrt{u(\beta-1)}\left[\frac{x}{2cos\theta} - arccos\frac{\varphi-2u}{a}\right]\right) - 1} \tag{24}$$

$$\varphi = \frac{-(a+2u)}{\frac{2a}{a-2u}ch^2\left(\sqrt{u(\beta-1)}\left[\frac{x}{2cos\theta} - arccos\frac{2u-\varphi}{a}\right]\right) - 1} \tag{25}$$

3.2 Small amplitude solitons. Comparison with the results of the reductive perturbation method

The rarefactive soliton amplitude is small if

$$\left(\frac{v_{||}}{v_0}\right)_{max} = a + 2u \ll 1 \tag{26}$$

It means that $2\sqrt{u}(sin\theta\sqrt{\beta} + \sqrt{u}) \ll 1$ or $u = 1 - \beta cos^2\theta \to 0$. So, $v_0 \approx C_A cos\theta$, the wave velocity is close to the Alfvén wave velocity. Small amplitude Alfvén soliton could be a rarefactive soliton.

Compressive soliton amplitude is small when

$$\left(\frac{v_{||}}{v_0}\right)_{max} = a - 2u \ll 1 \tag{27}$$

or $2\sqrt{u}\frac{\beta-1}{sin\theta\sqrt{\beta}+\sqrt{u}} \ll 1$. It means that there are two situations: $v_0 \to C_A cos\theta$ and $v_0 \to C_A$. So, in the small amplitude approximation eq. (24) describes one compressive soliton otherwise eq. (25) describes two rarefactive solitons, Alfvén and magnetosonic ones.

From the other hand, we can find the solution of the initial system of equations (1-7) using the reductive perturbation method in the form proposed in [11]. The initial system of equations is reduced to the KdV equation for the magnetosonic waves

$$\frac{\partial n}{\partial \tau_0} + \frac{3}{2}n\frac{\partial n}{\partial \xi_0} - \frac{ctg^2\theta}{2}\frac{\partial^3 n}{\partial \xi_0^3} = 0 \tag{28}$$

where $\tau_0 = \varphi^{\frac{3}{2}}t'$, $\xi_0 = \varphi^{\frac{1}{2}}(z'-t')$, variables $z' = \frac{\omega_{Hi}}{C_A}z$, and t' are dimensionless, and to the mKdV equation for the Alfvén waves

$$\frac{\partial n}{\partial \tau_0} + \frac{3}{4}ctg^2\theta n^2\frac{\partial n}{\partial \xi_0} + \frac{ctg^2\theta}{2}\frac{\partial^3 n}{\partial \xi_0^3} = 0 \tag{29}$$

where $\tau_0 = \varphi^{\frac{1}{2}}t'cos\theta$, $\xi_0 = \varphi^{\frac{3}{2}}(z' - t'cos\theta)$. The solutions of these equations respectively are [10]

$$n = 1 - sech^2\left[\frac{\varphi^{\frac{1}{2}}tg\theta}{2}\left(z - \left(1 - \frac{\varphi}{2}\right)t\right)\right] \tag{30}$$

$$n = 1 \pm sech\left[\frac{\varphi}{2cos\theta}\left(z - \left(1 + \frac{1}{8}\varphi^2 ctg^2\theta\right)t\right)\right] \tag{31}$$

Using approximation (27), it is possible to obtain relation (30) from (25) for the magnetosonic soliton, introducing soliton velocity in the form

$$\frac{v_0}{C_A} = 1 - \frac{\varphi}{2}.$$

In the same manner, using approximations (26) and (27), it is possible to obtain the relation (31) from (24) and (25) for Alfvén solitons, introducing soliton velocity in the form

$$\frac{v_0}{C_A} = cos\theta\left(1 + \frac{1}{8}\varphi^2 ctg^2\theta\right).$$

It should be mentioned that it is possible to obtain the following automodel equation for the hot plasma by the method of undetermined coefficients

$$(A\varphi_t + B\varphi_{ttt} + 2C\varphi) + D\varphi_{ttttt} + E\varphi_t\varphi_{tt} + G\varphi\varphi_{ttt} = 0 \qquad (32)$$

where t is automodel variable, the coefficients A, B, C, D, E, G are omitted because they are too cumbersome. In the case of cold plasma, with $D = E = G = 0$, eq.(32) coincides with eq.(28) in the automodel form. Comparison of eq. (32) for the cold plasma limit with the result of the exact solution is out of the scope of the present work.

Fig. 2(a-c) present the dependence of the rarefactive soliton half width, $\bar{L} = \frac{L}{C_A\omega_{Hi}^{-1}}$, on the amplitude for the different angles. The solid curve is calculated using exact formula (25), another curves are calculated using approximate formulae (30) and (31) for MsW and Alfvén solitons respectively. For the rarefactive soliton at the intermediate angles $\theta = 45^0$ (fig. 2b) results of the exact calculation and the results obtained by the reductive perturbation method (RPM) are in good agreement for Alfvén soliton in the whole range of possible amplitudes, even out of RPM application range [10] in the case if a non-dimensional amplitude is larger than 0.3. For the small values of angle $\theta = 10^0$ (fig. 2a) the divergence between the exact solution and RPM results both for Alfvén and MsW solitons is quite significant even in the small amplitude range, $\varphi \approx 0.015$, where RPM should work correctly. This happens because the amplitude value $\varphi \approx 0.015$ corresponds to the point of confluence of both exact solution branches. For the large angles θ we observe the coincidence of the exact solution and RPM results only in a small range of possible amplitudes, but this region includes large amplitude values $\varphi \approx 0.5$.

Dependence of soliton velocity on amplitude $\varphi = a - 2u$ for the rarefactive soliton is presented in fig. 2(d-f). For the intermediate and large angles Alfvén soliton velocity calculated using the RPM $\frac{v_0}{C_A} = cos\theta\left(1 + \frac{1}{8}\varphi^2 ctg^2\theta\right)$, (see formula 31), is in good agreement with the exact solution practically in the whole range of possible amplitudes for the intermediate and large angle $\theta \approx 45^0$ (fig. 2e) and $\theta \approx 80^0$ (fig. 2f).

RPM and exact results for the compressive soliton are in good agreement in an unexpectedly wide range of amplitudes, up to $\varphi \approx 1$ for $\theta = 45^0$ and even $\varphi \approx 5$ for $\theta = 80^0$.

Dependence of soliton magnetic field components $\tilde{h}_x + sin\theta$, $\tilde{h}_y$ and amplitude φ on automodel variable ξ for the different values of velocity v_0 and angle value $\theta = 45^0$ is presented in fig. 3 for compressive and rarefactive solitons (a-c and d-f respectively). Figures are plotted for the different soliton velocity values. In fig. 3a (velocity value $v_0 \approx C_A cos\theta$) there are small amplitude variations $\delta\varphi \approx 0.1$ and comparatively large variations of the magnetic field, $\delta\tilde{h}_x \approx 1$. While increasing velocity (fig.3b, c)) soliton tends to be more narrow and sharp, its amplitude increases. Fig. 3c is plotted for the velocity close to the critical velocity $v_0 \approx v_c$, where v_c is calculated in the point $\varphi = 1$. As could be seen from eq.(4), in this case the density is very high, $\frac{n}{n_0}(\xi = 0) \approx 300$. Soliton velocity v_0 is close to C_A on the fig. 3d. One could notice small amplitude variations, $\delta\varphi \approx 0.1$, and significant magnetic field variations, $\delta\tilde{h}_x \approx 1.2$, in contrast to the fig. 3f, $v_0 \approx C_A cos\theta$, where the amplitude variations are comparable to those on top plot but magnetic field variations are comparatively small, $\delta\tilde{h}_x \approx 0.1$. For the intermediate soliton velocity $C_A cos\theta < v_0 < C_A$, see fig. 3e, variations are large for both amplitude $\delta\varphi \approx 0.4$ and magnetic field $\delta\tilde{h}_x \approx 0.8$.

3.3 Nonlinear periodical structures

Let us consider periodical structures $i \neq 0$ ($i = 0$ corresponds to the case of the solitary wave). As seen from fig. 1a, three types of structures exist. The phase plane is separated by the separatrixes representing the solitary solutions. When the solution is close to the solitary (small i), the structure is highly nonlinear. On the other hand, if i increases, the structure tends to decrease its nonlinearity.

The structure of (a) type is significantly nonlinear (fig 4a). It could have only large amplitude (the region outside the outer separatrix in fig. 1a) and its presentation in the Fourier domain is wide, comparatively to the other types. Being considered as a density variation, it presents a periodical sequence of compressive pulses. The region of its existence consists of regions I and II in fig. 1b. The possible amplitude value for (a) type structures is limited from the upper side by the condition $\varphi < 1$ ($n = \frac{n_0}{1-\varphi}$) which represents the density positiveness condition.

Type (b) is the intermediate between the types (a) and (c). The structure is rarefactive when the solution goes near the inner separatrix and compressive near the outer separatrix. Dependence of nonlinear structure magnetic field components $\tilde{h}_x + sin\theta$, $\tilde{h}_y$ (dashed curves) and amplitude φ (solid curve) on automodel variable ξ is presented on fig. 4b. For the larger values of i it represents a small amplitude nonlinear oscillations. The region of existence for the structures of this type is presented by the region III in fig. 1b.

Type (c) represents quasi-harmonic rarefactive wave (fig. 4c). The region of its existence is presented by region II in fig. 1b.

It is necessary to mention that the structures of described types of structures are of significantly large amplitude. They are absent in the small amplitude approach.

4 Conclusions

In the present work we consider oblique nonlinear magnetosonic and Alfvén waves of large amplitude in cold magnetized plasma. The obtained equations (14-16) completely describe the one-dimensional ULF waves in the automodel approach: two possible types of solitons, compressive and rarefactive, and three types of nonlinear structures.

The exact solutions of the nonlinear MHD equations (14-16) were compared with the results obtained by the Reductive Perturbation Method. It was shown that in some specific cases RPM and exact solutions are in good agreement in unexpectedly wide region of amplitudes, for example up to amplitude values $\varphi \approx 2$ ($\theta = 45^0$) and $\varphi \approx 5$ ($\theta = 80^0$) for compressive solitons. The obtained solutions of the nonlinear equations could be useful for the verification of new analytical methods of derivation of the evolutional equations with higher nonlinear and diffraction terms.

We stress that the obtained analytical description of the ULF waves in the cold plasma (eq. 14-17), is simple and suitable for the qualitative analysis. It could be used for the interpretation of results in space experiments and numerical simulations.

5 Acknowledgment

This work was carried out with the partial support of the National Space Agency of Ukraine (contract N 6-103/96).

References

[1] Sagdeev, R.Z., Journal of Technical Physics **31**, 1185 (1961).

[2] Kellog, P.J., Phys. Fluids **7**, 1555 (1964).

[3] Tverskoj, B.A., Zh. Eksp. Teor. Fiz. (Sov. Phys. JETP) **42**, 833 (1962).

[4] Pataraja, A.D., Zh. Eksp. Teor. Fiz. (Sov. Phys. JETP) **32**, 139 (1962).

[5] Sherwell, D., and Cairnsr, A.J., Plasma Phys. **17**, 265 (1976).

[6] Danilov, A.V., Zh. Eksp. Teor. Fiz. (Sov. Phys. JETP) **101**, 89 (1992).

[7] Priest, E.R., "Solar Magnetohydrodynamics" (D. Reidel Publishing company,Dordrecht, Holland, 1982).

[8] Lembege, B., Ratliff, S.T., Dawson, J.M. and Ohsawa, Y., Phys. Rew. Lett. **53**, 1053 (1984).

[9] Start, D.F., Nucl. Fusion **30**, 2170 (1990).

[10] Kakutani, T., Suppl. Progr. Theor. Phys. **55**, 97 (1974).

[11] Janaki, M.S., Som, B.K. and Dasgupta, M.R., J. Phys. Soc. Japan **60**, 2977 (1991).

[12] Mikhailovsky, A.B., Aburdjanija, G.D. and Onischenko, O.G., Zh. Eksp. Teor. Fiz. (Sov. Phys. JETP) **89**, 482 (1985).

[13] Janaki, M.S., Dasgupta, B., Gupta, M.R and Som, B.K., Physica Scripta **45**, 368 (1992).

[14] Mio, K., Ogino, T., Minami, K., and Takeda, S., J. Phys. Soc. Japan **41**, 265 (1976).

[15] Yadav, L.L., Tiwari, R.S. and Sgarma, S.R., J. Plasma Phys. **51 (part 3)**, 355 (1994).

[16] Kaufman A.N. and Stenflo, L., Physica Scripta **11**, 269 (1975).

[17] Stenflo, L., Shvartsburg, A.B. and Weiland, J., Phys. Lett. A **225**, 113 (1997).

[18] Coleman, P.J., J. Geophys. Res. **71**, 5509 (1966).

[19] Belcher, J.W. and Davis, L., Jr., J. Geophys. Res. **76**, 3534 (1971).

[20] Denskat, K.U. and Neubauer, F.M., J. Geophys. Res. **87**, 2215 (1982).

[21] Galeev, A.A., Klimov, S.I., Nozdrachev, M.N., Sagdeev, R.S. and Sokolov, A.Yu., Zh. Eksp. Teor. Fiz. (Sov. Phys. JETP) **90**, 1690 (1986).

[22] Wu, C.S., Physica Scripta **60**, 91 (1995).

[23] Wu, C.S., Wang, D.Y. and Huang, G.L., Phys. Plasmas **4**, 611 (1997).

[24] Stenflo, L. and Shukla, P.K., Physica Scripta **48**, 370 (1993).

[25] Shukla, P.K. and Stenflo, L., Physica Scripta **T 60**, 32 (1995).

[26] Pokhotelov, O.A., Pokhotelov, D.O., Gokhberg, M.B., Feygin, F.Z., Stenflo, L. and Shukla, P.K., J. Geophys. Res. **101**, 7913 (1996).

Figure 1: (a) Phase plane ($\tilde{h}_y$ depending on $\tilde{h}_x$). Soliton solutions are marked with the bold line. (b) Regions of existence of nonlinear structures depending on parameter $i = c - \alpha^2$ and velocity v_0/C_A for angle $\theta = 30^0$. Region I corresponds type (a) structures, region II to types (a) and (b), region III to type (c). No periodical structures are in region IV. $i = 0$ corresponds to soliton.

Figure 2: (a-c) Rarefactive soliton half width dependence on the amplitude $\varphi = a - 2u$ for the different angles: (a) - 10^0, (b) - 45^0, (c) - 80^0. (d-f) Rarefactive soliton velocity dependence on amplitude φ for the different angles: (d) - 10^0, (e) - 45^0, (f) - 80^0. Solid curve corresponds to the exact solution. Bold and thin dashed curves correspond to Alfvén and magnetosonic solitons respectively.

Figure 3: Dependence of soliton magnetic field components $\tilde{h}_x + sin\theta$, $\tilde{h}_y$ (dashed curves) and amplitude φ (solid curve) on automodel variable ξ for the different values of velocity v_0 and angle value $\theta = 45^0$ for compressive (a-c) and rarefactive (d-f) solitons.

Figure 4: Dependence of nonlinear structure magnetic field components $\tilde{h}_x + sin\theta$, $\tilde{h}_y$ (dashed curves) and amplitude φ (solid curve) on automodel variable ξ for the type (a), (b) and (c) structures and angle value $\theta = 45^0$.